NONISOTHERMAL COUÉTTE FLOW AND PHASE TRANSITIONS IN A VISCOUS FLUID

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In the flow of a thermosensitive fluid, dissipative heat release leads to critical thermal phenomena dependent on the type of flow and the rheological properties of the fluid [1]. Often the fluid flow is accompanied by a phase or structural transition in a narrow temperature region. Some applications of these processes include transport of paraffin-base petroleum along a pipe [2], viscosimetric flow of a liquid crystal polymer [3], motion of magma with respect to a dike [4, 5], deformation of a liquid layer in the slip zone of contact with a solid [6, 7], etc. The combined effect of dissipative heat release and a phase transition leads to critical thermal phenomena. Thus there is interest in critical thermal phenomena in the presence of a phase transition, including the determination of the critical conditions, and stability, the nonuniqueness of the solution, the nature of unsteady processes following the loss of stability, and so on. These questions have not been discussed in the literature. They can be treated as Stefan problems [8] with the inclusion of dissipative heat release.

In the present paper, we consider the effect of a phase transition on hydrodynamic thermal explosions. It is known that for nonisothermal Couétte flow in a viscous fluid, this effect occurs when there is a constant stress on the moving boundary [9]. The nonuniqueness of the stationary states is demonstrated and the nature of their stability is discussed. Analytical expressions are obtained for two types of critical conditions: the limit of a thermal explosion and the condition for the establishment of a phase boundary inside the layer. Ranges of parameters are found which divide the qualitatively different flow regimes: a stationary regime and unsteady thermal explosion and complete phase transition regimes.

Interesting features occur in the treatment of a phase transition for a constant velocity flow. In this case a unique and stable stationary state in possible with an arbitrary position for the phase boundary depending on the intensity of the dissipative heat release.

The practically important question of the interpretation of viscosimetric experiments in the presence of a phase transition is discussed.

1. Statement of the Problem. We consider the flow of a viscous fluid between two infinite parallel plates $x = 0$ and $x = h$, where the plate $x = 0$ moves along the y axis. Dissipative heat release in the liquid is taken into account, and the dependence of the viscosity η on temperature T is taken in the form $\eta = \eta_0 \exp(E/RT)$. Here η_0 is a constant, E is the activation energy of viscous flow, and R is the universal gas constant. The plate $x = 0$ is assumed to be thermally insulated. On the other plate, the temperature is suddenly lowered to a value T_0 below the phase transition temperature T_* and subsequently held constant. Because of supercooling ($\Delta T = T_0 - T_*$) a first-order phase transition with a release of heat occurs. Because of the outflow of heat, the front dividing the phases moves away from the plate $x = h$.

Mathematically, the problem reduces to solving the equations:

$$T > T_*\colon\ c_1\rho_1 \frac{\partial T}{\partial t} = \lambda_1 \frac{\partial^2 T}{\partial x^2} + \sigma_{xy}\frac{\partial v_y}{\partial x},\quad \frac{\partial v_y}{\partial t} = \frac{1}{\rho_1}\frac{\partial \sigma_{xy}}{\partial x},\quad \sigma_{xy} = \eta\frac{\partial v_y}{\partial x}; \tag{1.1}$$

$$T < T_*\colon\ c_2\rho_2\frac{\partial T}{\partial t} = \lambda_2\frac{\partial^2 T}{\partial x^2}. \tag{1.2}$$

The boundary conditions for the heat equation are

$$x = 0\colon\ \partial T/\partial x = 0;\ x = h : T = T_0; \tag{1.3}$$

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$$x = x_*\colon\ T = T_*,\quad Q_0\rho_1 \frac{\partial x_*}{\partial t} + \lambda_1 \frac{\partial T}{\partial x}\bigg|_{x=x_*-0} = \lambda_2 \frac{\partial T}{\partial x}\bigg|_{x=x_*+0}. \tag{1.4}$$

Below we will consider separately the case of a specified stress and a specified velocity on the moving boundary:

$$x = 0\colon\quad \sigma_{xy} = \sigma_0;\quad x = x_*\colon\quad v_y = 0; \tag{1.5}$$

$$x = 0\colon\quad v_y = v_0;\quad x = x_*\colon\quad v_y = 0. \tag{1.6}$$

In equations (1.1) through (1.6), v_y and σ_{xy} are the velocity and stress, c_1, c_2, ρ_1, ρ_2, λ_1, λ_2, are respectively the heat capacities, densities, and thermal conductivities of the two phases, x, t are the spatial coordinate and the time, x_* is the position of the phase front, and Q_0 is the heat of the phase transition. The initial temperature distribution is not specified, and we study below the stationary problem, which using the dimensionless variables

$$\theta = E(T - T_*)/RT_*^2,\quad \xi = x/h,\quad v = v_y/v_0,\quad \sigma = \sigma_{xy}/\sigma_0$$

and the Frank–Kamenetskii approximation [10], takes the form

$$\theta > 0\colon\ \theta'' + \delta c^2 \exp\theta = 0,\ [v' \exp(-\theta)]' = 0; \tag{1.7}$$

$$\theta < 0\colon\ \theta'' = 0; \tag{1.8}$$

$$\xi = 0\colon\ \theta' = 0;\ \xi = 1\colon\quad \theta = \theta_0; \tag{1.9}$$

$$\xi = \xi_*\colon\ \theta = 0;\quad \lambda\theta'_+ = \theta'_-. \tag{1.10}$$

Here $\lambda = \lambda_2/\lambda_1$; $\theta_0 = E(T_0 - T_*)/RT_*^2$, θ'_+ θ'_- are the limiting values of θ' to the right and left of the point $\xi_* = x_*/h$. Boundary conditions (1.5) and (1.6) for the velocity take the form

$$\xi = 0\colon\quad \sigma = v' \exp(-\theta) = 1;\quad \xi = \xi_*\colon\quad v = 0; \tag{1.11}$$

$$\xi = 0\colon\quad v = 1;\quad \xi = \xi_*\colon\quad v = 0. \tag{1.12}$$

The parameter δ (see (1.7) and (1.8)) has a form dependent on the boundary conditions:

$$\delta = v_0 E \eta_0 (\lambda_1 R T_*^2)^{-1} \exp(E/RT_*) \tag{1.13}$$

for a specified velocity (1.12); and

$$\delta = h^2 \sigma_0^2 E (\eta_0 \lambda_1 R T_*^2)^{-1} \exp(-E/RT_*) \tag{1.14}$$

for a specified stress (1.11). From (1.7), the temperatures, heat fluxes, and velocities are determined as

$$\theta = \theta_0 (\xi - \xi_*)/(1 - \xi_*),\quad \theta' = \theta_0/(1 - \xi_*) \tag{1.15}$$

in the region $\xi_* < \xi < 1$, and

$$\theta = \ln \frac{a}{\mathrm{ch}^2\left(b + \sqrt{a\delta c^2/2}\,\xi\right)},\quad \theta' = -2\sqrt{a\delta c^2/2}\,\mathrm{th}\left(\sqrt{a\delta c^2/2}\,\xi + b\right); \tag{1.16}$$

$$v = \sqrt{2a/\delta}\left[\mathrm{th}\left(\sqrt{a\delta c^2/2}\,\xi_* + b\right) - \mathrm{th}\left(\sqrt{a\delta c^2/2}\,\xi + b\right)\right] \tag{1.17}$$

in the region $0 < \xi < \xi_*$ (see [11, 12]). Here a, b, c are constants of integration and depend on the type of deformation.

2. Specified Stress. We obtain b = 0 and c = 1 from the first boundary condition of (1.9) and (1.11), respectively. The third constant and the position of the phase front are determined from the second condition of (1.9) and (1.10). We have the relations:

$$\mathrm{ch}^2\left(\sqrt{a\delta/2}\,\xi_*\right) = a; \tag{2.1}$$

$$2s/(1 - \xi_*) = \sqrt{2a\delta}\,\mathrm{th}\left(\sqrt{a\delta/2}\,\xi_*\right). \tag{2.2}$$

Here and below we use the notation $s = \lambda\theta_0/2$. From (2.1) we have for ξ_*

$$\sqrt{\delta/2}\,\xi_* = (1/\sqrt{a}) \ln\left(\sqrt{a} + \sqrt{a - 1}\right) \tag{2.3}$$

and from (2.2) and (2.3) we can obtain the relation

$$\sqrt{\delta/2} = (1/\sqrt{a}) \ln\left(\sqrt{a} + \sqrt{a - 1}\right) + s/\left(\sqrt{a - 1}\right). \tag{2.4}$$

Equation (2.4) together with (1.15), (1.16), (1.17), and (2.3) can be considered a parametric solution of the problem. Thus for any values $a > 1$ and $s > 0$, there exists a unique δ and a corresponding solution ξ_*, θ. Noting that $\delta \to \infty$ for $a \to 1$ and $\delta \to 0$ for $a \to \infty$, we conclude that for any $\delta > 0$, $s > 0$, there exists a solution.

Relation (2.2) is analogous to a heat balance equation: The left-hand side is the rate of heat loss q_- for the region $\xi < \xi_*$ and the right-hand side is the rate of heat release q_+ in this region. In the quasistationary approximation, when the thermal and hydrodynamic relaxation times are much less than the characteristic phase transition time (i.e., a large heat of transition), the equation of motion for the phase boundary (see (1.4)) takes the form

$$\frac{d\xi_*}{d\tau} = q_+ - q_-, \quad \tau = t\frac{\lambda_1 RT_*^2}{h^2\rho_1 Q_0 E} \tag{2.5}$$

and connects the stationary characteristics q_+, q_- with the dynamics of the phase front.

We note that to each value of ξ_* there correspond two values of $q_+(\xi_*)$. However, there is only a lower branch in (2.5). The upper value of q_+ must be discarded because it corresponds to an unstable temperature distribution as in the heat explosion problem of [10] at fixed ξ_*. Also as the phase transition develops (ξ_* decreases), the rates of heat gain q_+ and heat loss q_- decrease nonlinearly (see Fig. 1).

When the parameter δ changes, q_- remains the same and only q_+ changes. In a particular range of δ (for sufficiently small $s < s_*$; see below) the curves q_+ and q_- intersect at three points and these correspond to the stationary values of ξ_*: lower ξ_{*1}, intermediate ξ_{*2}, and upper ξ_{*3}. According to (2.5), the lower ξ_{*1} and upper ξ_{*3} values are unstable, since physically ξ_* decreases for $q_+ < q_-$ and increases for $q_+ > q_-$. Thus only the intermediate value ξ_{*2} is stable.

When $\xi < \xi_{*1}$ the rate of heat loss exceeds the rate of heat release and according to (2.5), ξ_* goes to zero. This means the phase transition is complete. In this case the difference $q_- - q_+$ increases and hence ξ_* moves with an increasing velocity. Thus one can speak of a progressive freezing. We note that in the classical Stefan problem, this effect is impossible since there is no heat release ($\delta = 0$).

If $\xi_* > \xi_{*3}$, $q_+ - q_- > 0$ and the phase boundary moves toward the colder wall: $\xi_* \to 1$. Thus for the initial condition $\xi_*|_{\tau=0} > \xi_{*3}$ (see (2.5)) the stable stationary state $\xi_* = \xi_{*2}$ does not occur and this is also true for the initial condition $\xi_*|_{\tau=0} < \xi_{*1}$.

When δ decreases, the values ξ_{*1} and ξ_{*2} at the intersection of q_+ and q_- approach and join at a critical value $\delta = \delta_1(s)$ which separates a steady regime with a phase front inside the layer from the unsteady development of a complete phase transition. This critical condition also occurs in the case of a constant heat flux ($\theta > 0$; $\theta'' + \delta = 0$).

When δ increases, ξ_{*3} and ξ_{*2} approach and join at an upper critical value $\delta = \delta_2(s)$ which separates a steady regime with a phase front inside the layer from the unsteady progressive self-heating of the fluid, called a hydrodynamic heat explosion [13]. The critical value of $\delta_2(s)$ is caused by the nonlinearity of the heat source.

To find the critical values δ_1 and δ_2, we turn to the $\delta(\xi_*)$ dependence given in (2.3) and (2.4). We note that the mathematical formulation of the problem with a specified stress is completely analogous to the problem of a chemical reaction occurring in the presence of a

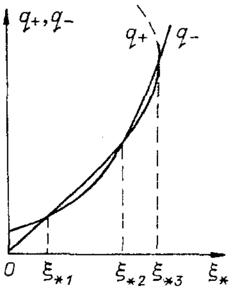

Fig. 1

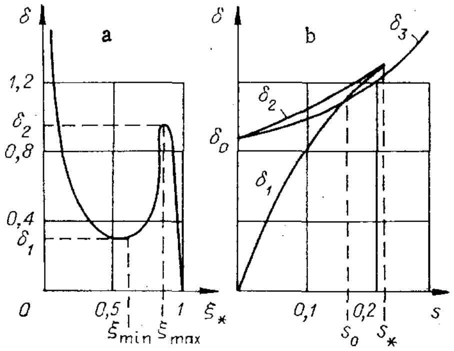

Fig. 2

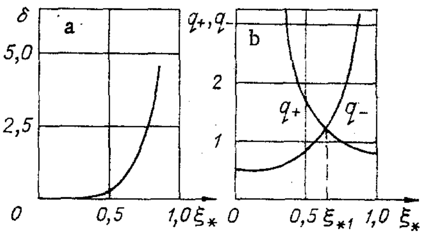

Fig. 3

phase transition [14] and the calculation of the critical values given in [14] can be used in the present context.

For a certain interval of s, the $\delta(\xi^*)$ curve is nonmonotonic and hence for a particular value of δ the solution is not unique (see Fig. 2a). In this case the boundary of the region where the solution is not unique is given by $0 < s < s_*$, $\delta_1(s) < \delta < \delta_2(s)$, where δ_1, δ_2 are the extreme values of $\delta(\xi_*)$ and $s_* \approx 0.21$ (see [14]). The functions $\delta_1(s)$ and $\delta_2(s)$ are found approximately as asymptotic expansions in s:

$$\delta_1 = 8s(1 - (2/3)s - \ldots); \tag{2.6}$$

$$\delta_2 = \delta_0(1 + 2s + \ldots), \quad \delta_0 = 0.878. \tag{2.7}$$

In Fig. 2b we show the stationary region for the system. Figure 2a shows the $\delta(\xi_*)$ curve in the region where the solution is not unique. Approximate asymptotic expressions valid for small values of s can be obtained [14] for $\xi_{\min}$ and $\xi_{\max}$:

$$\xi_{\min} = (1/2)(1 + (2/3)s + \ldots); \tag{2.8}$$

$$\xi_{\max} = 1 - s - \ldots \tag{2.9}$$

As s increases, δ_1 and δ_2 approach one another and join for $s > s_*$. The curve $\delta(\xi_*)$ becomes a monotonic decrease and the solution is now unique for any δ. As was shown above, the stationary state on the boundary of the nonunique region is unstable. It is not difficult to see from the physical meaning of ξ_* or from an analysis of the quasistationary equation (2.5) that any stationary state corresponding to the decreasing part of the $\delta(\xi_*)$ curve is unstable. Therefore, in the region $s > s_*$ there are no stationary stable states and the above analysis on the possible cases does not apply here. Nevertheless, one naturally expects that in this region complete phase transition and hydrodynamic heat explosion regimes also occur and that there is a critical value $\delta = \delta_3(s)$ separating the two regimes. However, this question cannot be answered in the stationary approach.

By analyzing the quasistationary equation (2.5), which is also defined for $s > s_*$, it can be shown that for values of the parameters where a stationary state is absent, a complete phase transition occurs and ξ_* goes to zero. For values of the parameters such that these exists an unstable stationary state, the phase front ξ_* (for the initial condition $\xi_* = 1$ at $\tau = 0$) cannot go to zero and a progressive self-heating of the fluid takes place.

The boundary $\delta_3(s)$ of these regions can be found from the condition that the stationary points of (2.5) vanish or go to the upper branch of the curve $q_+(\xi_*)$: $q_+ = q_-$, $d\xi_*/dq_+ = 0$.

Using (2.1) through (2.3), the system of equations can be rewritten in the form

$$2s(1 - \xi_*)^{-1} = [2\delta(a - 1)]^{1/2}; \tag{2.10}$$

$$\ln(\sqrt{a} + \sqrt{a - 1}) = \sqrt{a}/\sqrt{a - 1}. \tag{2.11}$$

Equation (2.11) is well known from the theory of heat explosions. The solution for $2/(a - 1)$ is 0.878 or $a = 3.28$. We obtain for $\delta_3(s)$

$$\delta_3 = 0{,}878(1 + s)^2. \tag{2.12}$$

In Fig. 2b we show a diagram of the regions corresponding to qualitatively different regimes in the plane defined by the parameters δ and s. In the region $\delta > \delta_3(s)$, $s \geqslant 0$, we have a hydrodynamic heat explosion in spite of the fact that in part of this region there are stable stationary states. The initial value $\xi_* = 1$ for $\tau = 0$ is outside of the region. In the region $\delta_1(s) < \delta < \delta_3(s)$, $s < s_0$, the stationary value ξ_* is established inside the layer; this is calculated from (2.3) and (2.4). Then the intersection s_0 of the curves δ_1 and δ_3 can be found easily: $s_0 \approx 0.168$. In the region $\delta < \delta_1(s)$, $s < s_0$ or $\delta < \delta_3(s)$,

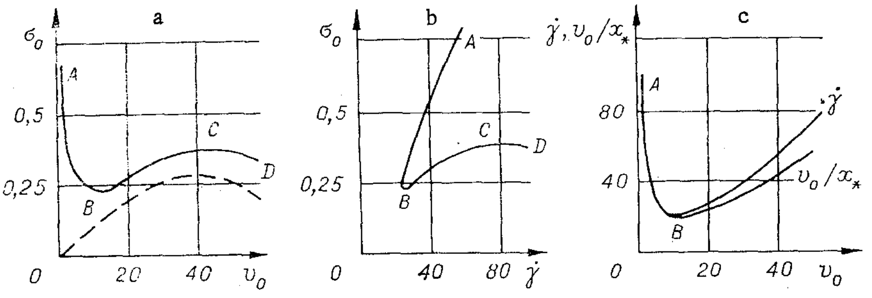

Fig. 4

$s > s_0$, a complete phase transition takes place independently of the initial value ξ_*.

It should be emphasized that this analysis assumes a restriction on the initial temperature distribution. In the viscous case it must be such that the above quasistationary approximation can be used. Thus the temperature gradient cannot be too large, which would predetermine an explosive process and the temperature cannot be too close to T_*, where thermal processes are slowed or do not exist.

3. Specified Velocity. From the boundary condition (1.9), it follows at once that $b = 0$. In order to determine the other two integration constants and the position of the phase transition front we have from the second condition of (1.9), (1.10)

$$\operatorname{th}\left(\sqrt{a\delta c^2/2}\,\xi_*\right) = -\sqrt{\delta/2a}, \quad \operatorname{ch}^2\left(\sqrt{a\delta c^2/2}\,\xi_*\right) = a; \tag{3.1}$$

$$2s(1-\xi_*)^{-1} = 2\sqrt{a\delta c^2/2}\,\operatorname{th}\left(\sqrt{a\delta c^2/2}\,\xi_*\right). \tag{3.2}$$

Solving these relations, we get

$$\begin{gathered} a = \delta/2 + 1, \quad c = \frac{2s}{\delta(1-\xi_*)}, \\ \xi_*(\delta) = \frac{f(\delta)}{f(\delta)+s}, \quad f(\delta) = \frac{1}{2}\sqrt{\frac{\delta}{\delta+2}}\ln\left(\frac{\sqrt{\delta+2}+\sqrt{\delta}}{\sqrt{\delta+2}-\sqrt{\delta}}\right). \end{gathered} \tag{3.3}$$

Equation (3.2), as was shown above, is analogous to a thermal balance equation. On the right-hand side is the rate of heat release, and on the left, the rate of heat loss. It can be shown easily that (3.3) is a monotonic increase such that $\xi_* \to 0$ as $\delta \to 0$ and $\xi_* \to 1$ as $\delta \to \infty$ (see Fig. 3a, $s = 0.2$). Thus for each $\delta > 0$ there exists a unique ξ_*.

As in Sec. 2, it is convenient to analyze the dependence of the rate of heat release q_+ and rate of heat loss q_- on the phase transition parameter ξ_* (see Fig. 3b, $s = 0.2$, $\delta = 1$). It turns out that for all values there exists a unique intersection point ξ_{*1}, corresponding to a stationary solution. If we take into account that the direction of motion of the phase transition boundary depends on the difference between q_+ and q_-, then we see that for any perturbation taking ξ_* to the left or right of ξ_{*1}, the system returns to the stationary state, so that the state is stable.

It is important to note that the rate of heat release increases with the extent of motion of the phase transition front, and $q_+ \to \infty$ for $\xi_* \to 0$. This is because when ξ_* decreases, the velocity gradient $\dot{\gamma} = dv_y/dx$ progressively increases. Because of this effect, in the course of the phase transition the dissipative heat release increases, and the moving phase transition front is slowed down and finally becomes fixed inside the gap. It then follows that a complete phase transition is impossible (this is also an important feature of constant velocity flow) due to the dependence of the dissipative heat release on the rate of shear ($q_+ \sim \dot{\gamma}^2$). For example, if we have a constant source of heat release ($\theta'' + \delta = 0$, $\theta > 0$) it is easy to show that there is a critical condition $\delta_* = 8s$ (solution of the problem gives $\delta = 2s\xi_*^{-1}(1-\xi_*)^{-1}$), below which a stationary solution does not exist and a complete phase transition occurs.

The establishment of stable stationary states with any possible position of the phase transition front depending on the velocity of the moving boundary is important in the application of dissipative heat release in directional crystallization, processes which cannot be attained using high-frequency heating [15].

4. Rheological Curve in the Presence of a Phase Transition. An important practical problem is the determination of the rheological characteristics in the presence of a phase transi-

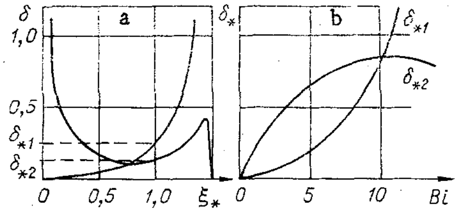

Fig. 5

tion. From (2.1) and (2.2) one can obtain $\sigma_0(v_0)$ and $\sigma_0(\dot{\gamma})$ using the following relations:

$$\sigma_0 = \frac{2\eta(T_*)}{h\delta}[s + f(\delta)],\ \dot{\gamma} = \left.\frac{dv_y}{dx}\right|_{x=0} = \frac{\sigma_0}{\eta(T_*)}\left(\frac{\delta}{2} + 1\right), \tag{4.1}$$

where δ is determined by (1.12). For comparison, in Fig. 4a, the dashed line shows the qualitative form of the rheological curve in the absence of a phase transition. It is clear from Fig. 4b that the viscosimetric curve $\sigma_0(v_0)$ and rheological curve $\sigma_0(\dot{\gamma})$ differ qualitatively from each other. Hence a rheological treatment of the measurements in the presence of a phase transition must be done with care. In the case of constant shear (σ_0 = const), only branch BC is truly rheological in the sense that it is stable and the viscosity is correctly determined on it. In the constant velocity case (v_0 = const), all branches of the rheological curve are stable; however in determining the viscosity on branch AB one must take into account the large depth of the phase transition and on branches BC and CD, the additional effect of self-heating of the fluid.

It is known that in a rotating viscosimeter, the velocity gradient $\dot{\gamma}$ is determined in terms of the velocity on the moving boundary v_0 and the separation of the two boundaries h. In particular, for the planar model used here, we have the formula $\dot{\gamma} = v_0/h$. However, from the dependence $\dot{\gamma}(v_0)$ in Fig. 4c, it follows that this formula is not valid when there is a phase transition. In the nearly isothermal case for small enough v_0 (branch AB of Fig. 4c) the phase front moves to a large depth ($\xi_* < 0.5$) and therefore the thickness of the liquid phase decreases. In this case one expects the formula

$$\dot{\gamma} = v_0/\xi_* \tag{4.2}$$

to be valid. Then the viscosity at the temperature of the isothermal wall $T = T_0$ is given by

$$\eta = \sigma_0\xi_*/v_0. \tag{4.3}$$

The validity of (4.2) and (4.3) along branch AB is supported by the numerical results: The velocity gradient up to the minimum point on Fig. 4c agrees with the theoretical values, and the viscosity remains constant along AB, which agrees with the assumptions made here.

5. Effect of Heat Exchange. If on the plate x = h, a boundary condition of the third kind is satisfied for the heat equation, then the second relation of (1.9) takes the form

$$\xi = 1{:}d\theta/d\xi = -\mathrm{Bi}\,(\theta - \theta_0),\ \mathrm{Bi} = \alpha h/\lambda_1.$$

For the case of a specified velocity we have for the constants of integration:

$$a = \delta/2 + 1,\quad b = 0,\quad c = \frac{2}{\delta}\,\frac{s}{1 - \xi_* + \mathrm{Bi}^{-1}}$$

and for a specified stress we have

$$a = 1 + \frac{2}{\delta}\left[\frac{s}{1 - \xi_* + \mathrm{Bi}^{-1}}\right]^2,\quad b = 0,\quad c = 1.$$

The position of the phase transition front in the specified velocity case (compare with (3.3)) is given by

$$\xi_*(\delta) = \left(1 + \mathrm{Bi}^{-1}\right) f(\delta)/(f(\delta) + s). \tag{5.1}$$

When the stress is specified, we obtain the parametric form (compare with (2.3) and (2.4))

$$\delta = \left[\frac{\sqrt{2/a}\,\ln\left(\sqrt{a} + \sqrt{a-1}\right) - \sqrt{2}\,s/\sqrt{a-1}}{1 + \mathrm{Bi}^{-1}}\right]^2, \tag{5.2}$$

$$\xi_* = \sqrt{\frac{2}{a\delta}}\,\ln\left(\sqrt{a} + \sqrt{a-1}\right).$$

In Fig. 5a (Bi = 2, s = 0.05) we show the curves $\delta(\xi_*)$ for a specified velocity and stress. It follows from (5.1) that $\delta(\xi_*)$ for the specified velocity case has the asymptotic form $\xi_* = 1 + \mathrm{Bi}^{-1}$, and part of the curve can be outside of the interval $0 < \xi_* < 1$. Hence there is a critical value δ_{*1} such that for $\delta > \delta_{*1}$ a phase transition does not take place because the power of the dissipative heat release is too large. In the case of a specified stress, part of the curve $\delta(\xi_*)$ also can lie outside of the interval $0 < \xi_* < 1$ and similarly there is a phase transition threshold δ_{*2}. In the interval $0 < \xi_* < 1$ (depending on Bi) there can be only a decreasing branch $\delta(\xi_*)$ corresponding to an unstable stationary state. In Fig. 5b we show the curves $\delta_{*1}(\mathrm{Bi})$ and $\delta_{*2}(\mathrm{Bi})$ for the specified stress and specified velocity cases.

The critical conditions for a hydrodynamic heat explosion and complete phase transition can be represented in terms of the parameters:

$$\bar{\xi}_* = \xi_*(1 + \mathrm{Bi}^{-1})^{-1}, \quad \bar{\delta} = \delta(1 + \mathrm{Bi}^{-1})^2.$$

Relation (5.2), when the new parameters $\bar{\xi}_*$ and $\bar{\delta}$ are used, takes the form (2.3) and (2.4) and all of the results in Sec. 2 for the critical values δ_1, δ_2, δ_3 and extremal points $\xi_{\min}$, $\xi_{\max}$ (see (2.6) through (2.12)) remain true for the boundary condition of the third kind on the fixed plate.

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